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Draining from rapidly spinning tubes

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When a liquid-filled tube of radius b which is spinning about its axis with angular velocity Ω is allowed to drain by opening one end, a cavity propagates in the tube with velocity U. This velocity has been measured with the tube in both vertical and horizontal positions for large values of the ratio $\Omega^2 b/g$ of the centrifugal to the gravitational force, g being the gravitational acceleration. It is found that the Rossby number $U/\Omega b$ becomes constant and independent of $\Omega^2 b/g$ in both cases, taking the value $U/\Omega b = 0{\cdot}52$ in the vertical case and $U/\Omega b \simeq 0{\cdot}4$ in the horizontal case. Experiments conducted in a tube which is slightly inclined to the horizontal confirm that, although the Rossby number is constant when $\Omega^2 b/g$ is large, the value adopted depends upon α, the inclination of the tube to the gravitational field.

1. Introduction

The arrangement considered here consists of a long cylindrical tube of cross-sectional radius b which is filled with water and made to spin steadily about its axis with angular velocity Ω. Some time after the start of rotation when the water has acquired a rigid-body rotation, the tube is opened at one end so that the water then drains from it while an air-filled cavity propagates in the tube with velocity U. On the assumption that both viscous and surface-tension forces are unimportant compared with inertia forces, dimensional analysis suggests that

$$f_1(U/(gb)^{\frac{1}{2}}, U/\Omega b, \alpha) = 0, \tag{1}$$

where α is the inclination of the tube axis to the vertical and g denotes the gravitational acceleration. Useful alternative forms of (1) involving the dependent variable U in only one of the groups are

$$U/(gb)^{\frac{1}{2}} = f_2(\Omega^2 b/g, \alpha) \tag{2}$$

and

$$U/\Omega b = f_3(\Omega^2 b/g, \alpha). \tag{3}$$

The behaviour of this system when the tube is vertical and opened at its lower end, that is with $\alpha = 0$, has been studied experimentally and theoretically in an earlier paper which will be referred to subsequently as I (Collins & Hoath 1973). There, it was found that a simple approximate theoretical model provided excellent descriptions of the development of the motion with increasing values of $\Omega^2 b/g$. This group expresses the ratio of centrifugal forces to gravitational forces and is the independent parameter which was varied in the experiments by changing Ω. With no rotation, the theory predicted constant values of the Froude number $Fr = U/(gb)^{\frac{1}{2}}$ which were found to be close to experimental values and when $\Omega^2 b/g$ was greater than about 12, the experimental system adopted a value of the Rossby number $Ro = U/\Omega b$ which remained essentially constant at $Ro = 0{\cdot}52$. The latter finding, which agreed with the theory, showed that, when centrifugal forces dominate gravitational forces, the cavity propagates with a velocity equal to the maximum group velocity of infinitesimal waves, which is $c_0 = 0{\cdot}52\ \Omega b$.

When paper I was being refereed, two referees raised questions concerned with the interpretation of the experimental results. The authors had argued on dimensional grounds that a constant value of the Rossby number could be anticipated when $\Omega^2 b/g$ was large and that the limiting value was effectively reached in the experiments when $\Omega^2 b/g > 12$. This conclusion was supported by the theory. The referees, however, emphasized the fact that earlier experiments in a *horizontal* tube performed by Benjamin & Barnard (1964) had shown the limiting value to be significantly lower, at $Ro \simeq 0{\cdot}38$.† Benjamin & Barnard had also invoked a dimensional argument to show that the Rossby number would be expected to become constant for high values of $\Omega^2 b/g$ and their own experiments had covered the range $41 < \Omega^2 b/g < 138$, whereas those in the vertical tube employed a maximum value $\Omega^2 b/g = 27$ and showed the Rossby number to be constant in the range $12 < \Omega^2 b/g < 27$. The referees suggested that ultimately, for large values of $\Omega^2 b/g$, the two systems must be expected to behave in the same manner, adopting the same constant value of the Rossby number, which would thus be independent of the tube inclination α. They pointed out that the maximum value of $\Omega^2 b/g$ employed in the experiments in the vertical tube was below the minimum used by Benjamin & Barnard in the horizontal system and suggested that, although the Rossby number was essentially constant over the range $12 < \Omega^2 b/g < 27$, the experiments in the vertical tube did not describe the correct asymptotic behaviour. Paper I had naturally referred to Benjamin & Barnard's experiments as providing an interesting contrast but since, in the present authors' view, the two systems were essentially different because of their differing inclinations to the gravitational field, it had not been anticipated that they should produce identical numerical values for the limiting Rossby number. In formulating a reply to the referees it became clear that it was not possible to resolve these questions with the data then available because, when these were plotted in the form shown in figure 6‡ of I, an additional question concerning the constancy of Benjamin & Barnard's data arose. It is possible, for example, to interpret their data as showing the Rossby number to increase with increasing $\Omega^2 b/g$, possibly reaching a limiting value common with that from the vertical tube. Regrettably, the apparatus built for I had not been designed for safe operation above a value of $\Omega^2 b/g = 27$,

† In I, the sentence beginning in line 23 on page 517 should read: 'Moreover, since $\alpha \neq 0$ in their model, ...'.

‡ It is convenient to use $g/\Omega^2 b$ as the abscissa so as to facilitate extrapolation to $g/\Omega^2 b = 0$. For the vertical tube many more experimental points showing the development of the flow were obtained for values of $g/\Omega^2 b$ outside the range shown in this figure.

so that it was not possible to use it in order to extend the range. Accordingly in the final draft of I the precise nature of the asymptotic behaviour was left as an open question which might be resolved by further experimentation. In the experiments reported here, the range of $\Omega^2 b/g$ covered by the vertical system has been extended so as to embrace the values previously used by Benjamin & Barnard, more experimental points have been obtained for the horizontal system to supplement Benjamin & Barnard's and some results have been obtained for one position in which the tube was inclined at a slight angle to the horizontal.

2. Apparatus

In the apparatus used in I, the limitation on speed was imposed by the nature and spacing of the bearings which supported the tube. The major modification has thus been to remount the Perspex tube of length 1·865 m and internal diameter 65 mm in four substantial ball races spaced equally along the tube. With this arrangement, the first critical speed when the tube was filled with water was calculated to be well above the maximum speed of approximately 34 rev/s employed in this series of tests. In non-dimensional terms, the force ratio $\Omega^2 b/g$ achieved a maximum value of 158, which is somewhat above the maximum in Benjamin & Barnard's experiments. Cavity speed was again measured by allowing the cavity nose to interrupt two light beams a known distance apart, but the method of measuring rotational speed was changed. A magnetic transducer monitored the rotation of a steel gear wheel having sixty teeth which rotated with the tube and the speed was determined by counting the pulses induced by the passage of these teeth over consecutive periods of 1 s. The result was displayed on a digital meter. Some modifications to the rotating removable stopper were also carried out. Instead of making the seal with an O-ring seated on the inner curved surface of the tube as in I, a face-seal on the end of the tube was used. The stopper was held concentric with the tube by three small cylindrical pins projecting into the tube interior and bearing on the inner surface of the wall.

Still photographs were again taken to reveal the general development of cavity shape with increasing $\Omega^2 b/g$ but the cavity velocity was too high to allow a crisp image to be obtained by photographing against a background of diffused light as before. Instead, flash photography with side illumination was used.

3. Discussion

All data on cavity velocity now appear in figures 1 and 2. As far as the vertical tube is concerned, the more recent experimental points are seen to be entirely consistent with those in I and are significantly less scattered. This characteristic is attributed to the modified arrangement of the bearings, which contributes greater stiffness to the Perspex tube and also permits a more precise control over its alignment. The experimental points show that the constant Rossby number achieved in this case is $Ro = 0{\cdot}52$, as given in I, and that this value is

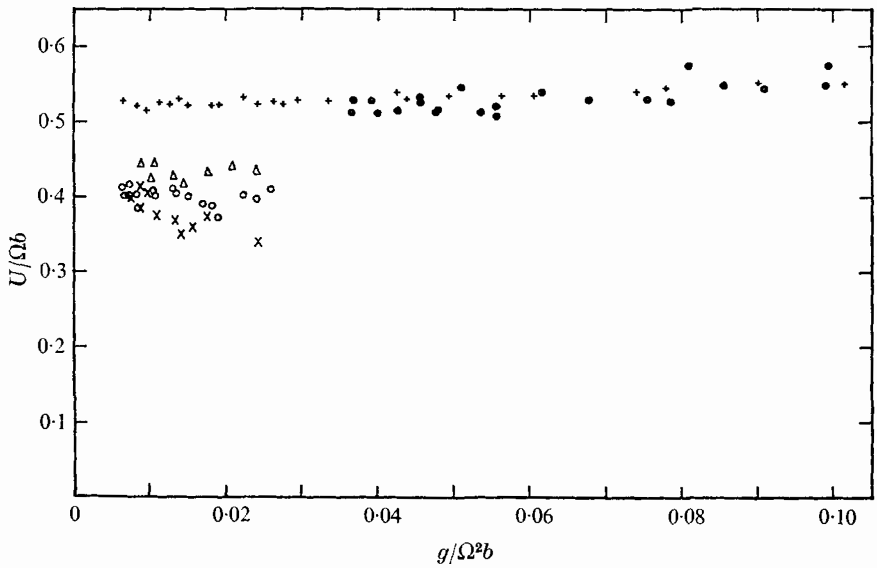


FIGURE 1. Dependence of Rossby number on $g/\Omega^2 b$. $\alpha = 0$: ●, Collins & Hoath (1973); +, this work. $\alpha = \frac{1}{2}\pi$: ×, Benjamin & Barnard (1964); ○, this work. $\alpha = 77°$: △, this work.

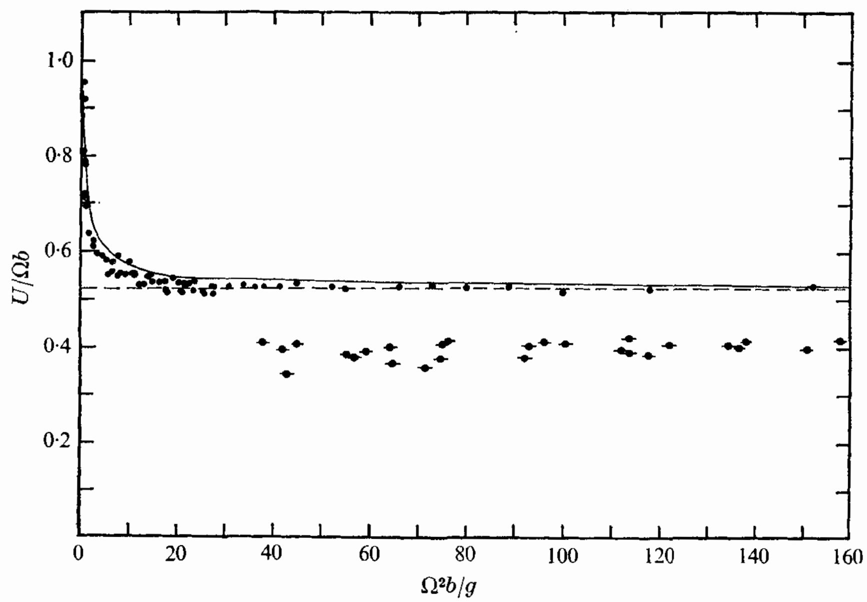


FIGURE 2. Dependence of Rossby number on $\Omega^2 b/g$. ●, all data available for $\alpha = 0$; -●-, all data available for $\alpha = \frac{1}{2}\pi$; ———, single-term approximate theory for $\alpha = 0$, Collins & Hoath (1973); – – – –, $U/\Omega b = 0{\cdot}52$.

sensibly attained when $\Omega^2 b/g > 12$. The cavity velocity is thus seen to be equal to the maximum group velocity of infinitesimal waves in the rotating system, which is $c_0 = 0{\cdot}52\ \Omega b$.

The newer experimental points for the horizontal tube, where $\alpha = \frac{1}{2}\pi$, are rather more scattered than those obtained when the tube is vertical. They are in reasonable agreement with Benjamin & Barnard's results since they suggest that the Rossby number is substantially constant at $Ro \simeq 0{\cdot}4$, slightly above Benjamin & Barnard's value of $Ro \simeq 0{\cdot}38$. We observe that neither the newer data for the horizontal tube nor the data for a tube inclination of $\alpha = 77°$, which show $Ro \simeq 0{\cdot}43$ in figure 1, exhibit any pronounced tendency for the Rossby number to increase with increasing $\Omega^2 b/g$. It may be that such an interpretation of Benjamin & Barnard's data for the horizontal tube attaches too much significance to the experimental point corresponding to the lowest value of $\Omega^2 b/g$ which they used. There can be little doubt over the constant nature of the Rossby number achieved in the vertical tube and although the somewhat larger scatter of points in the experiments for the horizontal tube does not instil quite the same confidence for that case, the appropriate interpretation of figures 1 and 2 appears to be that, when $\Omega^2 b/g \gg 1$, the Rossby number becomes independent of $\Omega^2 b/g$ but remains a function of the inclination α. That is to say, the magnitude of the gravitational force is dominated by the magnitude of the centrifugal force in these circumstances to the extent that the influence of the former on the motion becomes insignificant, but its direction relative to the axis of rotation remains a pertinent variable. Figures 1 and 2 show no discontinuous behaviour for the vertical system over the range tested (a possibility suggested by a referee of I was that the results for the vertical tube would fall discontinuously to reproduce those for the horizontal tube) and if we make the usual assumption on extrapolation to the effect that no discontinuities occur outside the range, then extrapolation in figure 1 to the limit $g/\Omega^2 b = 0$ would suggest that the limiting value of the Rossby number remains a function of the inclination α. Results obtained by extrapolation are of course always of uncertain status.

In assessing the situation revealed by the experiments, the problem is first to decide what behaviour it is reasonable to anticipate in the horizontal and vertical systems, particularly when $\Omega^2 b/g \gg 1$. The experimental results may then be viewed in the light of these expectations. Consider two separate systems, which will be identified by a suffix 1 or 2, and consider the conditions of operation required so that one system is a dynamically similar model of the other. A prerequisite for the achievement of dynamic similarity is that the two systems be geometrically similar and in the free boundary problem under consideration geometric similarity involves similarity of the inclination of the apparatus to the gravitational field. The first condition to be fulfilled is thus $\alpha_1 = \alpha_2$, and if in addition $(\Omega^2 b/g)_1 = (\Omega^2 b/g)_2$ then, from (3), dynamic similarity of the two systems will exist and $(U/\Omega b)_1 = (U/\Omega b)_2$. It is worth noting that these considerations do not involve any restriction on the magnitude of the force ratio $\Omega^2 b/g$ nor do they invoke any assumptions about the particular form of the relationship between $U/\Omega b$ and $\Omega^2 b/g$. Since equality of tube inclinations is a necessary condition for dynamic similarity, a vertical system cannot model a horizontal system whatever the magnitude of $\Omega^2 b/g$, so that in terms of (3) it is concluded that we should anticipate that

$$f_3(\Omega^2 b/g, 0) \neq f_3(\Omega^2 b/g, \tfrac{1}{2}\pi). \tag{4}$$

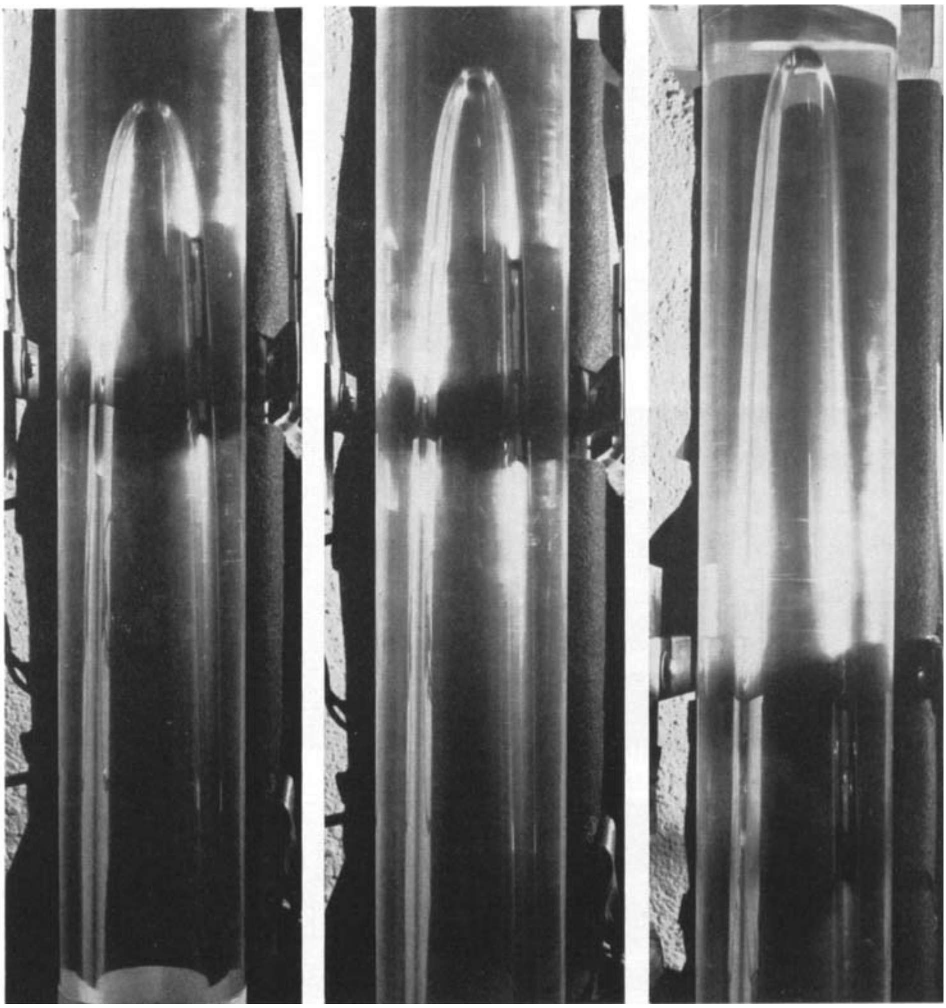
Consideration of the nature of the equations governing the two flows leads to the same result. It is expected, then, that the Rossby number will remain a function of α for all values of $\Omega^2 b/g$. Further, for $\Omega^2 b/g \gg 1$, when centrifugal forces dominate gravitational forces, it is reasonable to expect that the dependence on $\Omega^2 b/g$ will become very weak, so that the Rossby number will be essentially constant for a given α. The experimental results are in agreement with these expectations.

The only circumstance in which the problems become mathematically identical is when $g \equiv 0$ for then there is no acceleration along the vertical tube, no asymmetry in the horizontal tube and the cavity boundary condition which will ensure that the cavity is a constant-pressure surface becomes $q = 0$, where q is the magnitude of the local velocity on its surface. It was for this idealized situation that Benjamin & Barnard proved that a mathematical solution of the problem which had flow upstream steady in a frame of reference moving with the cavity was impossible. Their experiments in a horizontal tube in fact showed the existence of a Taylor column moving ahead of the cavity with velocity c_0. Their analysis, which by its nature could not describe any dependence on α, led them to conclude that $Ro \ngtr 0{\cdot}52$ when $g \equiv 0$. The trend of all experimental results in figure 1 agrees with this limitation.

Finally, figure 3 (plate 1) and figure 4 (plate 2) show the strikingly different cavity shapes which are observed when $\alpha = 0$ and when $\alpha = \frac{1}{2}\pi$. With $\alpha = 0$, the cavity surface is smooth and steady and the shape axisymmetric. With $\alpha = \frac{1}{2}\pi$, however, the surface is irregular and unsteady, the shape lacks axial symmetry and although the evidence is slender, in two of these photographs there may be a helical structure to the surface. Generally speaking, the present flash photographs for the horizontal tube taken with light reflecting from the slug surface show the surface to be rather more disordered than appears to be the case in the single photograph given by Benjamin & Barnard. Their photograph consisted of one frame from a cine film taken at 80 frames/s with the cavity viewed against a background of diffused light.

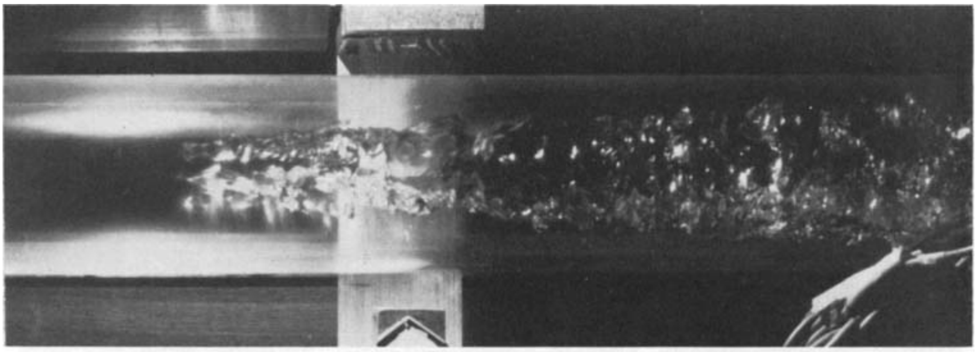
REFERENCES

Benjamin, T. B. & Barnard, B. J. S. 1964 *J. Fluid Mech.* **19**, 193.
Collins, R. & Hoath, M. T. 1973 *J. Fluid Mech.* **57**, 515.

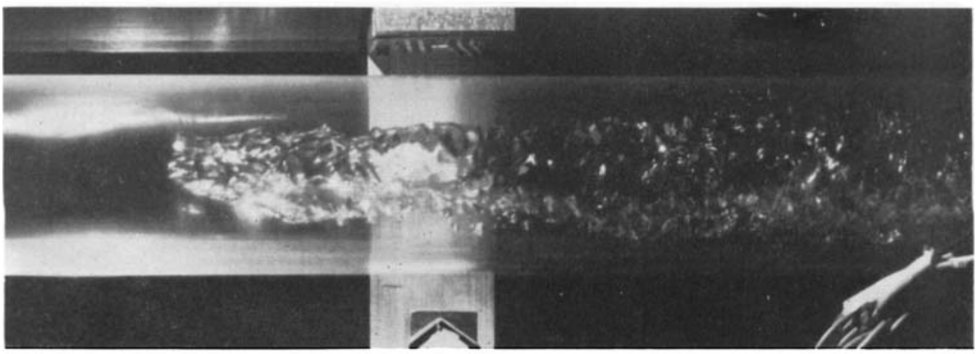


$\Omega^2 b/g = 37{\cdot}5$ $\Omega^2 b/g = 72{\cdot}5$ $\Omega^2 b/g = 105{\cdot}5$

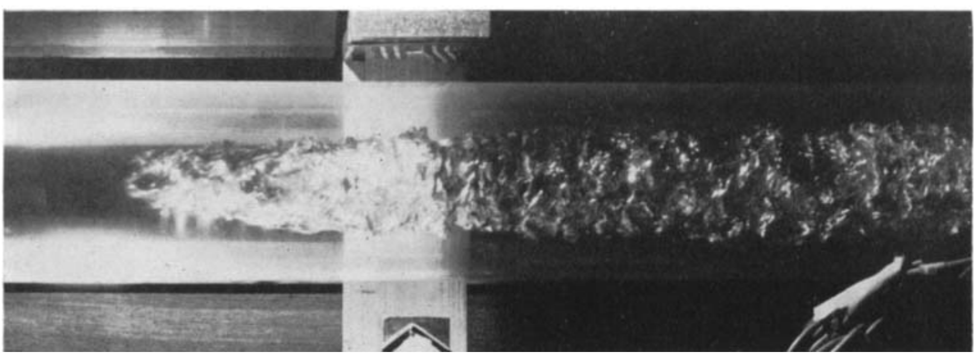
FIGURE 3. Photographs of cavity shape for various values of $\Omega^2 b/g$ when $\alpha = 0$.



$\Omega^2 b/g = 56$



$\Omega^2 b/g = 99$



$\Omega^2 b/g = 152$

FIGURE 4. Photographs of cavity shape for various values of $\Omega^2 b/g$ when $\alpha = \frac{1}{2}\pi$.